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Fuzzy decision making

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Abstract

In a decision making situation when one is faced with the problems of selecting a system from a given finite number of systems, one of the usual approaches is to seek opinions of different experts on these systems. Experts while expressing their opinions feel more comfortable and at ease if they are allowed to express their view point in an imprecise manner rather than exact manner. In this article, the decision making notion is studied in a vague situation with the aid of piecewise quadratic fuzzy numbers. A relevant practical example to illustrate theoretical ideas, is also included.

Keywords : decision making, fuzzy numbers, fuzzy sets, piecewise quadratic fuzzy numbers.

1. INTRODUCTION

Bellman and Zadeh (1970) have precisely defined the notion of decision making in a fuzzy environment. According to them, 'Decision making in a fuzzy environment is a decision process in which the goals and / or the constraints, but not necessarily the system under control, are fuzzy in nature. This means that goals and / or the constraints constitute classes of alternatives whose boundaries are not sharply defined'. Fuzziness furnishes choices. One can pick and choose from the shades of gray that defines fuzziness. More degrees manifest more options. Fuzzy goals and fuzzy constraints can be defined precisely as fuzzy sets in the appropriate space of alternatives. This appropriate space of alternatives is the consensus of the experts. A fuzzy decision then may be realized as an intersection of the given goals and the constraints. A maximizing decision is defined as a point in the space of alternatives at which the membership function of a fuzzy decision attains its maximum value.

There are decision situations in which the information can be assessed not precisely in a quantitative form but may be in a qualitative one. This makes it necessary to use the linguistic approach. For example, when attempting to qualify phenomena instead of quantification, based on human perception, we often tend to use words in natural language rather than the numerical values. There may arise some special situations where the information may be unquantifiable due to its nature and thus it may be stated only in linguistic terms. In other cases, precise quantitative information may not be stated because either it is unavailable or the cost of its computation is too high. In such situations an approximate value may be accepted.

2. INGREDIENTS OF DECISION PROCESS

In the accepted approach to decision making, the principal ingredients of a decision process are

i. a set of alternatives

ii. a set of constraints on the choice between different alternatives and

iii. a performance function which associates with each alternative, the gain (or loss) resulting from the choice of that alternative.

When we view a decision process of decision making in a fuzzy environment, a different and more natural conceptual framework suggests itself. The most important feature of this framework is its symmetry with respect to goals and constraints. This symmetry contemplates upon removing the differences between goals and constraints and makes it possible to relate it in a relatively simple way the concept of a decision to those of the goals and constraints of a decision process.

Turning to the notion of a 'decision' we observe that, a decision is basically a choice or a set of choices drawn from the available alternatives. A fuzzy decision or simply a decision be defined as the fuzzy set of alternatives resulting from the intersection of goals and constraints. In this context we provide below a definition proposed by Bellman and Zadeh (1970).

2.1 Definition

Assume that we are given a fuzzy goal G and a fuzzy constraints C in a space of alternatives X. Then G and C combine to form a decision D, which is a fuzzy set resulting from intersection of G and C, and correspondingly $m_D = m_G \wedge m_C$ where μ is the corresponding membership function. The relation between G, C and D is given in Figure 1.

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Figure 1. Decision in a Fuzzy Environment

According to Bellman and Zadeh (1970), a broad definition of the concept of decision may be stated as

Decision = Confluence of Goals and Constraints

There are many facts in the theory of decision making in a fuzzy environment which require more thorough investigation. The way in which the goals and the constraints must be combined if the goals and constraints are of unequal importance or are independent, is a sensitive domain.

3. FUZZY NUMBERS

Many fuzzy sets representing linguistic concepts such as 'low', 'medium', 'high' and so on are employed to define states of a variable. Such a variable is known as a fuzzy variable. The relevance of fuzzy variables is that they facilitate gradual transitions between states and consequently possess a natural capability to express and deal with observation and measurement uncertainties. In the traditional sense computing involves manipulation of numbers and symbols. But in contrast humans employ mostly words in computing, reasoning and arriving at natural language or having the form of mental perceptions. A key aspect of computing with words is that it involves a fusion of natural languages and computation with fuzzy variables. The notion of a granule plays a vital role in computing with words. According to Zadeh (1996), 'granulation plays a key role in human cognition. For humans it serves as a way of achieving data comparison'.

Fuzzy sets which are defined on the set R of real numbers endows a special importance. Membership functions $\mu : R \rightarrow [0,1]$ clearly possess a quantitative meaning and may be viewed as fuzzy numbers provided they satisfy certain conditions. Initiative conceptions of approximate numbers or intervals such as 'numbers that are close to 5' or numbers that are around the given real numbers'. Such notions are essential for characterizing states of fuzzy variables.

These fuzzy numbers play an important role in many applications including fuzzy control, decision making, approximate reasoning and optimization. A fuzzy number is the fuzzy subset of the real line where highest membership values are clustered around a given real number. For a fuzzy number the membership function is monotonic on both sides of the central value. In what follows we provide the definition of a fuzzy number which is commonly accepted in the literature.

3.1 Definition (Zedeh, 1975)

A fuzzy subset A of the real line R with membership function

$\mu_A : R \rightarrow [0,1]$ is called a fuzzy number if

i. A is normal, i.e. there exist an element $x_0 \in A$ such that $\mu_A(x_0) = 1$

ii. A is fuzzy convex. i.e., $\mu_A(\lambda x_1 + (1 - \lambda)x_2) \geq \mu_A(x_1) \Lambda \mu_A(x_2) \; \forall \; x_1, x_2 \in R$ and $\forall \; \lambda \in [0 , 1]$

iii. μA is upper semi continuous and

iv. Supp. A is bounded where Supp $A = \{x \in R : \mu_A(x) > 0\}$

The above definition is the generalized form of fuzzy numbers, though they vary diversely. Although the triangular and trapezoidal shapes of membership functions are used most often for representing fuzzy numbers, other shapes may be preferable in some applications. Moreover membership function of fuzzy numbers need not be symmetric.

A fuzzy number A can be thought of as containing real numbers within an interval A having varying levels of presumption or degrees of membership from 0 to 1. Though we have a variety of fuzzy numbers, so far as this work is concerned we contemplate more upon piecewise quadratic fuzzy numbers. Before elaborating as to why we choose piecewise quadratic fuzzy numbers we furnish below the definition of the same.

4. PIECEWISE QUADRATIC FUZZY NUMBERS (Bojadziev and Bojadziev, 1997)

4.1. Definition

A piecewise quadratic fuzzy number A is a fuzzy number fully specified by 5-tuples $(a_1, a_2, a_3, a_4, a_5)$ such that $a_1 \leq a_2 \leq a_3 \leq a_4 \leq a_5$. It is defined as

$$\mu_A(x) = \begin{cases} \frac{1}{2(a_2 - a_1)^2}(x - a_1)^2 & \text{for } a_1 \leq x \leq a_2 \\ \frac{-1}{2(a_3 - a_2)^2}(x - a_3)^2 + 1 & \text{for } a_2 \leq x \leq a_3 \\ \frac{-1}{2(a_4 - a_3)^2}(x - a_3)^2 + 1 & \text{for } a_3 \leq x \leq a_4 \\ \quad (x - a_5)^2 & \text{for } a_4 \leq x \leq a_5 \\ 0 & \text{otherwise} \end{cases} \qquad (4.1)$$

A policy maker has in his / her mind fuzzy goals with piecewise quadratic membership functions, the goals can easily be transformed into a conventional crisp goals utilizing Kim and Whang's model (Kim and Whang, 1998).

In what follows we provide two types of bell shaped fuzzy numbers. The well known normal distribution in probability is defined as

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} \quad -\infty < x < \infty \quad (4.2)$$

Where m is the mean and s is the standard deviation of the distribution. This distribution is very important because many random variables in the applications are normal or approximately normal. The curve f(x) is bell shaped. It is symmetric about the mean m. As the standard deviation s gets smaller, the peak become higher. More specifically for $\sigma = \frac{1}{\sqrt{2\pi}}$ the peak is $(\mu,1)$.

To construct a fuzzy number from a normal distribution we set in (4.2), $\sigma = \frac{1}{\sqrt{2\pi}}$ which gives a function with maximum 1.

If we take a continuous function $\alpha = F_A(x)$, $a_1 \le x \le a_5$, $F_A(a_1) = F_A(a_5) = 0$, with one maximum $(a_3,1)$ where $F_A(x)$ is the membership function depicting the bell shaped fuzzy number A, then for $\sigma = \frac{1}{\sqrt{2\pi}}$.

we have $\alpha = F_A(x) = e^{-\pi(x-\mu)^2}, x \in (-\infty,\infty), x \in [0,1]$ (4.3)

where μ is the parameter which determines the shape of $F_A(x)$. μ can assume any value but the maximum of $F_A(x)$ is always 1, hence (4.3) represented a bell-shaped fuzzy number whose supporting interval $(-\infty,\infty)$ is unbounded.

Consider the fuzzy membership function $F_A(x)$ defined by (4.1) as shown in figure 2.

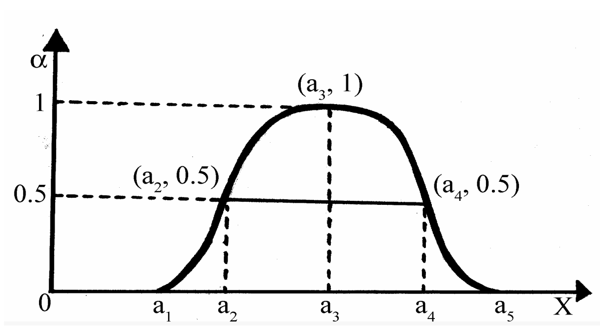

Figure 2. Piecewise Quadratic Fuzzy Number

5. APPLICATION

As a case study, let us consider the following Decision making problem in fuzzy environment. A special fuzzy decision making method (Nagoorani and Stephen Dinagar, 2005) has been utilized to solve the given problem. The steps taken in this problem are summarized as follows:

Step 1. Representation of the Decision Problem

(1) The decision goal is to select the most suitable option among the spent fuel storage options. Four decision alternatives are identified as follows:

A = {A1, A2, A3, A4}, where A1 = wet storage, A2 = dry horizontal modular storage, A3 = dry vault storage, and A4 = dry vertical concrete cask storage.

(2) Based on the previous study (Shin, 1994), the decision criteria are defined as follows:

C = {SA, EI, SI, CO, SR, FL, EN}, where SA = safety, EI = environmental impact, SI = socioeconomic impact, CO = cost, SR = site requirements,

FL = flexibility, and EN = engineering.

(3) The hierarchical structure of this problem is shown in Figure 3.

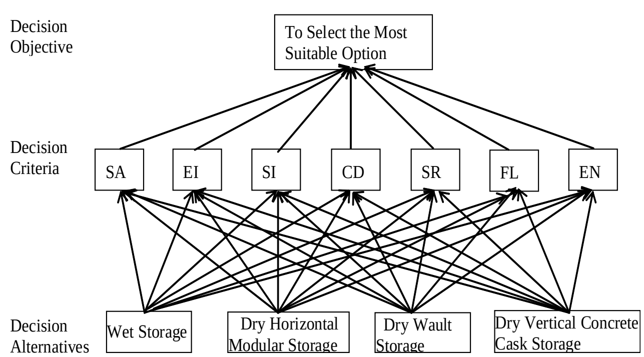

Figure 3. The Hierarchical structure

Step 2. Fuzzy Set Evaluation of the Decision Alternatives

(1) With the linguistic variables representing the importance weights of the decision criteria and the degree of appropriateness of the decision alternatives versus the decision criteria, the term sets are assigned as follows: T(importance) *W* = {VL, L, M, H, VH}, where VL = very low, L = low, M = medium, H = high, and VH = very high; and T(appropriateness) *S* = {VP, P, F, G, VG}, where VP = very poor, P = poor, F = fair, G = good, and VG =very good. The membership function corresponding to each element in each term set is

represented by the piecewise fuzzy number as defined in Nagoorani and Stephen Dinagar (2005).

2. Using the data in the previous study (Shin 1994), the ratings for the importance of each decision criterion and the degree of appropriateness of each decision alternative versus each decision criterion are assigned and summarized in Tables 1 and 2, respectively.

(3) Substituting the piecewise quadratic fuzzy numbers corresponding to each linguistic variable assigned to each decision criterion and alternative into equation, the fuzzy appropriateness indices for decision alternatives can be obtained. The results are summarized in the last column of Table 2.

Table 1. Importance rating for each decision making criterion

Criterion	SA	EI	SI	CO	SR	FL	EN
Importance Rating	VH	M	M	M	L	L	H

Step 3. Selection of the Optimal Alternatives

It is noticed that, regardless of optimism index value of the decision-maker, the rank of decision alternatives does not change. Hence, the wet storage (i.e., A1) is identified as the most suitable option for spent fuel interim storage. This is why the wet storage has been rated higher than other alternative versus the criteria with high importance such as safety, cost, and engineering.

Thus, a simple and easy-to-use method based on the fuzzy set theory was proposed to aid the evaluation of the decision alternatives with several ill-defined decision criteria. The applicability of this method was demonstrated through the example to select the most suitable option for the spent fuel storage.

6. CONCLUSION

Fuzzy set theory has become utmost relevance in most engineering areas particularly in decision making situations. This article addresses some notions and methodologies that have been proven useful in solving decision making problems. It also provides some techniques for decision making in fuzzy environment which include control and optimization methodologies.

Table 2. Appropriateness rating and fuzzy appropriateness index for each alternative

Alternative	Appropriateness Rating							Fuzzy Appropriateness Index
	SA	EI	SI	CO	SR	FL	EN	
A1	G	F	G	VG	G	F	G	(0.1429, 0.3304, 0.3929, 0.4554, 0.6429)
A2	G	F	F	F	F	G	F	(0.0625, 0.2500, 0.3125, 0.3750, 0.5625)
A3	G	G	F	VP	F	G	F	(0.0357, 0.2143, 0.2857, 0.3571, 0.5357)
A4	F	F	P	G	F	F	P	(0.0089, 0.1964, 0.2589, 0.3214, 0.5089)

Table 3. Total integral value and rank for various values of optimism index

	Optimism Index Value of Decision-maker		
	Moderate (=0.5)	Pessimistic (=0.0)	Optimistic (=1.0)
A1	0.3929 (1)	0.2887 (1)	0.4971 (1)
A2	0.3125 (2)	0.2083 (2)	0.4167 (2)
A3	0.2857 (3)	0.1786 (3)	0.3128 (3)
A4	0.2589 (4)	0.1547 (4)	0.3631 (4)

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