

## Time to breakdown point of an organization - Cumulative damage model

A. Yogeswari<sup>1\*</sup> and T. Chitrakalarani<sup>2</sup>

<sup>1</sup>Department of Mathematics, Sengamala Thayaar Educational Trust Women's College, Mannargudi - 614 016, Tamil Nadu, India.

<sup>2</sup>Department of Mathematics, K.N.G. Arts College for Women (Autonomous), Thanjavur, Tamil Nadu, India.

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### Abstract

This paper estimates the expected time to reach the uneconomic status of an organization due to successive occurrence of absenteeism of workers.

**Keywords:** absenteeism, break down point, cumulative damage, manpower planning, shock model

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### INTRODUCTION

Absenteeism is a social phenomenon, an industrial malady and a labour problem. Absenteeism refers to the workers absence from his regular task when he is normally scheduled to work. It results in loss of manpower to the organization. It leads to production losses in terms of quantity and quality and earnings are reduced to the organization.

Jensen and McIntosh (2006) developed a new model of absenteeism behaviour using Danish data from a worker environmental survey. Baladev (1970) examined the relationship between a number of demographic, social and organizational factors and absenteeism using primary data from automobile plant in Bombay. Froidevaux (1973) concludes that absenteeism is not just the result of employee sickness or fatigue but a sign of bad management. Enidperlin (1977) observes that high rate of absenteeism was noticeable on night shifts and in departments where conditions were most unpleasant. Desai (1970) gives analysis of attendance records, the sample survey and case studies of chronic absentees as the methods of organizing a research study on absenteeism. Coles and Treble (1993) extended a model developed by Wises (1985) in which the firms interest in reducing worker absenteeism is reflected in the wage contract it offers to its employees. Esary et al (1973) has considered Shock models and Wear processes. Dasgupta and Pecht (1991) have determined the material failure mechanisms and damage. Chitrakalarani and Ganesan (2002) have developed a stochastic model for the study of cumulative effects of

Industrial Accidents. Bhatia (1979) emphasizes the need for quantitative analysis and systematic understanding of the absenteeism problem.

The successive occurrence of absenteeism of workers leads to manpower loss to the organization. If the total loss of manpower exceeds the threshold level, the organization reaches the uneconomic status. The aim of this paper is to estimate the expected time to reach the uneconomic status of an organization due to successive occurrence of absenteeism of workers by using cumulative damage model.

### MODEL FORMULATION

#### Assumptions

- ◆ Absenteeism occurs at 'K' random epochs and at every epoch a random number of manpower loss leads to man hours loss to the organization.
- ◆ Each absenteeism epoch causes a random loss in man hours. That losses on successive absenteeism epochs are independently identically distributed (i.i.d) random variables.
- ◆ The process which generates the absenteeism epoch, the sequence of loss ' $X_i$ ' and the threshold level 'Y' are independent.
- ◆ If the total loss exceeds and threshold level 'Y' which itself is a random variable, the organization faces the breakdown point.

#### Notations

$X_i$  – A i.i.d continuous random variable that denotes the man hours lost due to the i-th absenteeism epoch.

Y – A continuous random variable that denotes the threshold level causing the organization reaches an

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\*Corresponding Author  
email: [yogibharathi@yahoo.com](mailto:yogibharathi@yahoo.com)

uneconomic status. Threshold level 'Y' has an exponential distribution with parameter 'θ'

T – A continuous random variable denoting the time to reach an uneconomic status of an organization.

G(x) – the cumulative distribution function of  $X_i = X$  for  $i = 1, 2, \dots, K$  and  $g(x)$  – corresponding probability density function.

$V_k(t)$  – Probability of exactly 'K' absenteeism epoch occurs in (0, t].

$U_i$  – A continuous random variable denoting the interarrival times between successive absenteeism epochs which has an exponential distribution with parameter C.

$F_k(t)$  – the distribution function of  $U_1 + U_2 + \dots + U_k$

$g_k(t)$  – probability density function of  $\sum_{i=1}^k X_i$

$g^*(\theta)$  – The laplace transform of  $g(x)$ .

**RESULTS**

The probability that the cumulative loss of man hour has not crossed the threshold level in 'K' absenteeism epochs is equal to  $S_k$  and.

$$P(T > t) = \sum_{k=0}^{\infty} P(\text{Exactly } k \text{ absenteeism epochs in } (0,t) \text{ and the threshold level is not crossed})$$

$$= \sum_{k=0}^{\infty} V_k(t) [g^*(\theta)]^k$$

$$= \sum_{k=0}^{\infty} (F_k(t) - F_{k+1}(t)) [g^*(\theta)]^k$$

$$= S(t) \text{ which is the survivor function}$$

$$L(t) = P(T > t)$$

$$= [1 - g^*(\theta)] \sum_{k=1}^{\infty} [1 - g^*(\theta)]^{k-1} F_k(t)$$

$$L^*(s) = [1 - g^*(\theta)] \sum_{k=1}^{\infty} [g^*(\theta)]^{k-1} (f^*(s))^k$$

$$L^*(s) = [1 - g^*(\theta)] f^*(s) \sum_{k=1}^{\infty} [g^*(\theta) f^*(s)]^{k-1}$$

$$= \frac{[1 - g^*(\theta)] f^*(s)}{[1 - g^*(\theta) f^*(s)]}$$

$$L^*(s) = \frac{[1 - \beta] [c/c + s]}{[1 - \beta (c/c + s)]} \text{ Where } \beta = g^*(\theta)$$

$$\text{Mean } \mu_1 = \frac{-d}{ds} [L^*(s)]|_{s=0}$$

$$= \frac{1}{c [1 - \beta]}$$

$$\text{Variance} = \frac{1}{c^2 [1 - \beta]^2}$$

**Special case**

If X follows exponential distribution with parameter α

$$g^*(\theta) = \frac{\alpha}{\alpha + \theta}$$

$$\text{Mean} = \frac{\alpha + \theta}{c\theta}$$

$$\text{Variance} = \frac{(\alpha + \theta)^2}{c^2\theta^2}$$

**Numerical illustrations :**

For variation in 'C' and 'θ' the numerical results are given in Table 1 and 2 which are supported by the graphs in Fig. 1 and 2.

**Table 1.** Analysis of mean

| C  | Mean    |         |         |         |         |
|----|---------|---------|---------|---------|---------|
|    | θ = 0.1 | θ = 0.3 | θ = 0.5 | θ = 0.7 | θ = 0.9 |
| 2  | 5.5000  | 2.1667  | 1.5000  | 1.2143  | 1.0556  |
| 4  | 2.7500  | 1.0833  | 0.7500  | 0.6071  | 0.8278  |
| 6  | 1.8333  | 0.7222  | 0.5000  | 0.4048  | 0.3519  |
| 8  | 1.3750  | 0.5417  | 0.3036  | 0.3036  | 0.2639  |
| 10 | 1.1000  | 0.4333  | 0.2429  | 0.2429  | 0.2111  |

**Table 2.** Analysis of variance

| C  | Variance |         |         |         |         |
|----|----------|---------|---------|---------|---------|
|    | θ = 0.1  | θ = 0.3 | θ = 0.5 | θ = 0.7 | θ = 0.9 |
| 2  | 30.2500  | 4.6944  | 2.2500  | 1.4745  | 1.1142  |
| 4  | 7.5625   | 1.1736  | 0.5625  | 0.3686  | 0.2785  |
| 6  | 3.3611   | 0.5216  | 0.2500  | 0.1638  | 0.1238  |
| 8  | 1.8906   | 0.2934  | 0.1406  | 0.0922  | 0.0696  |
| 10 | 1.2100   | 0.1878  | 0.0900  | 0.0589  | 0.0446  |

For fixed 'θ' as 'C' increases, the mean time to breakdown decreases. Also if 'C' is fixed and 'θ' increases, the mean time to breakdown decreases. The variance of time to breakdown is also numerically illustrated in table 2. For fixed value of 'θ' as 'C' increases the variance decreases. 'C' is fixed and θ increases again the variance decreases.

The overall conclusion that could be drawn from the behaviour of mean and variance of time to breakdown is that the number of absenteeism epoch when increased tends to shorten the time to breakdown.

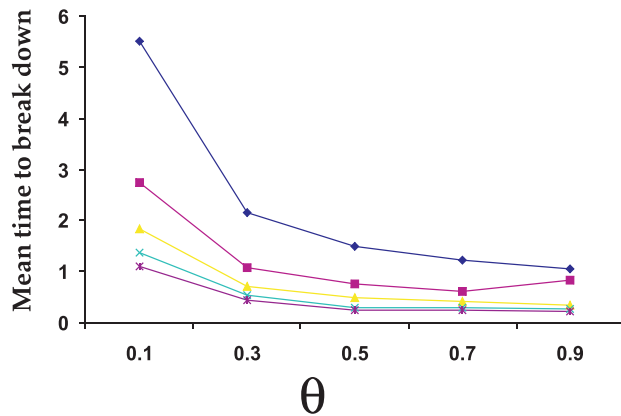


Figure 1. Behaviour of mean

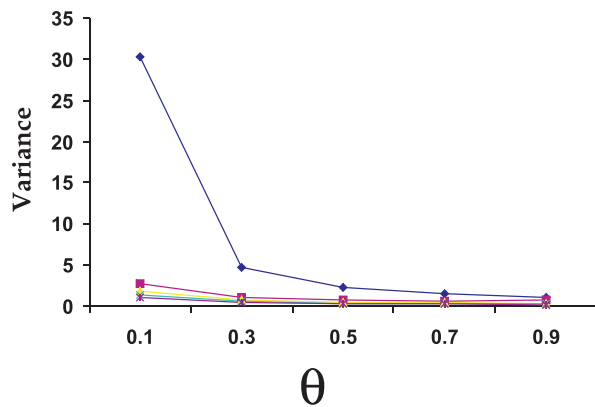


Figure 1. Behaviour of variance

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