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Application of gracefulness on V - tree

D.R.Kirubaharan* and B.Arunraj

Department of Mathematics, AVVM Sri Pushpam College (Autonomous) Poondi- 613 503. India.

Abstract

In this paper, we prove that V-tree* $2F_n$ is graceful.

Keywords: graceful graph, graceful labeling, graph theory, v-tree

INTRODUCTION

Graph theory is one of the most flourishing branches of mathematics with wide application to combinational problems. Graphs are usually represented by diagrams using a point for each vertex and a line for each edge. A graph G is an ordered triple of (V(G), E(G), ψ). If e is an edge and ψ(e) = (u, v), then we say that e is an edge joining u and v and the vertices u and v are called the ends of e. A graceful graph is a graph that has at least one graceful labeling. Now each edge is labled with the absolute difference of the endpoints of the concerned edge, the labeling is graceful, if the edges are labeled $1, 2, \ldots, n$ inclusive.

A graceful labeling on a graph with p vertices and q edges is a 1-1 map taking the vertices into the integers $0, 1, \ldots . q$ with the property that each edge uv is assigned by the label $|f(u)-f(v)|$.

A connected graph with n vertices and n-1 edges is called a tree, and that tree consists, root, stem, branch, leaf, blossom and flower.

The stem of the flower is the maximal common initial path and the blossom of the flower is the odd cycle obtained by deleting the stem and also a flower is the union of two *M*- alternating paths from u that a vertex x on steps of opposite parity.

In this paper we prove that V- tree$2F_n$ is graceful.*

Definition: 1

A *graceful labeling* of a graph G with m edges is a function $f: v(G) \rightarrow \{0,\ldots m\}$ such that, distinct vertices receive distinct numbers and $\{ |f(x) - f(y)| : xy \in E(G) \} = \{ 1,\ldots m\}$ and a graph is graceful, if it has a *graceful labeling*.

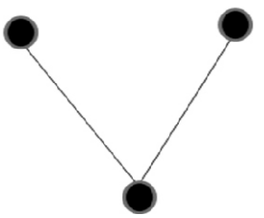
Classification of graceful tree

Definition: 2

A *leaf* (*Pendent vertex*) is a vertex of degree *1* and a graph is a *tree*, if it has at least one pendent vertex

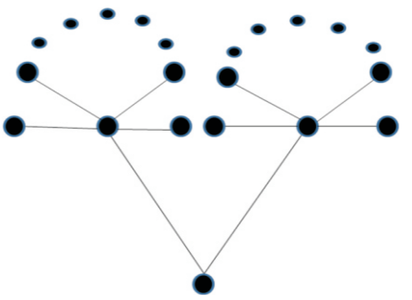
Definition: 3

A *V – tree* is defined as follows.



Definition: 4

The $V^* 2F_n$ flower (an *V-tree* combined with n stars in each of the two branches) is defined by the following tree.

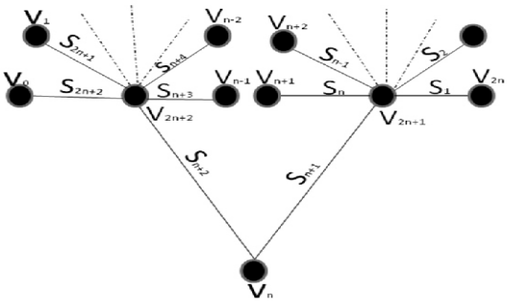


Theorem: 5

The *tree* $V^* 2F_n$ is graceful.

Proof:

Let the tree $V^* 2F_n$ be obtained by merging n copies of stars F_n with *V - tree,* which in given as bellow.



*Corresponding Author
email: jagkans@yahoo.com

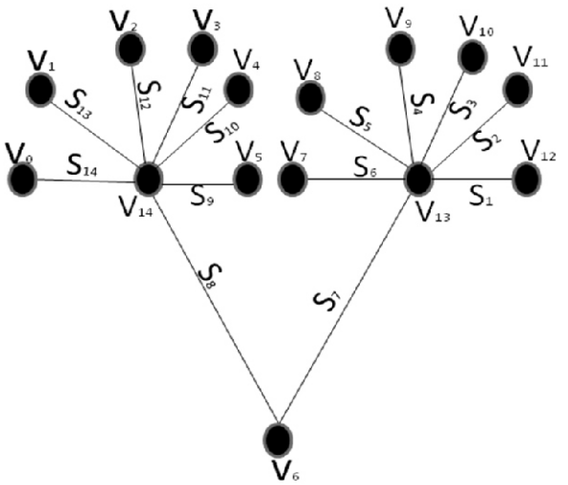
Let $p = 2n + 3$ be the number of vertices and $q = 2n + 2$ be the number of edges in the V^*2F_n - tree. Now, define a labeling f on the vertex set of V^*2F_n by $f(v_i) = i$ such that $i = 0$ to $2n + 2$ ……→ **1**

The edge set labeling e is defined by $e(uv) = |f(u) - f(v)|$, for any edge uv in the tree V^*2F_n.

Thus the given tree with the vertex labeling f and edge labeling e becomes a graceful tree. Hence V^*2F_n is a graceful tree.

Example:

Using the labeling in **1** the gracefulness of V^*2F_n is shown as below.



Let p = 15, q = 14 be the number of vertices and edges respectively and n = 6 be the number of stars merges in each branch of the *V- tree.* Define a vertex labeling 'f' on the V^*2F_6 by $F(v_i) = i$, $i = 0$ to 14 and edge labeling by $e(uv) = |f(u) - f(v)|$ for any edge uv in the tree, V^*2F_6.

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