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Fuzzy effective shortest spanning tree algorithm

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Abstract

In this paper, Fuzzy Effective Spanning tree, Fuzzy Effective branch, Fuzzy Effective chord, Fuzzy Effective rank, Fuzzy Effective nullity, Fuzzy Effective shortest spanning tree, Fuzzy Effective ring sum and Fuzzy Effective fundamental circuit are introduced and some interesting results for these new parameters in Fuzzy Graphs are obtained.

Keywords: fuzzy graph, fuzzy domination number, fuzzy effective distance, fuzzy effective rank, fuzzy effective nullity

INTRODUCTION

The first definition of fuzzy graph was proposed by Kafmann, from the fuzzy relations introduced by Zadeh. Although Rosenfeld introduced another elaborate definition, including fuzzy vertex and fuzzy edges, and several fuzzy analogs of graph theoretic concepts such as paths, cycles, and connectedness are introduced. Then after it many concepts such as fuzzy intersection graphs, fuzzy line graphs was introduced by Bhutani.and Rosefeld., 2003 a,b., The concepts of domination in fuzzy graphs were investigated by A.Somasundram, S.Somasundram and A.Somasundram present the concepts of independent domination, total domination, connected domination and domination in Cartesian product and composition of fuzzy graphs. In this paper we develop the concepts of fuzzy effective spanning tree.

The fuzzy effective spanning trees are more useful model to many real world problems. Suppose we want to locate a hospital, a police station, a fire station, radio station or any other such emergency service facility, we have to minimize the distance from the facility to the most remote place from at least one of the facilities. We can model this by a graph, where centers represent the vertices of the graph and the road link between centers represents the edges between the corresponding vertices. Such problems of locating emergency service facilities where we want to minimize the largest travel distance to any vertex from its nearest facility are called minmax location problem.

Preliminaries:

A Fuzzy subset of a nonempty set V is a mapping $\sigma : V \rightarrow [0,1]$.

A Fuzzy relation on V is a Fuzzy subset of V x V.

A Fuzzy graph is a pair of functions $\sigma : V \rightarrow [0,1]$ $\mu: V \times V \rightarrow [0,1]$ where for all u, v in V we have $\mu(u,v) \leq \min(\sigma(u), \sigma(v))$

A fuzzy graph $H : (\tau, \upsilon)$ is called fuzzy sub graph of $G: (\sigma, \mu)$ if $\tau(u) \leq \sigma(u)$ for all $u \varepsilon V$ and

$\upsilon(u, v) \leq \mu(u,v)$ for all u, $V \varepsilon V$.

A fuzzy graph $G = (V, \sigma, \mu)$ is called a strong fuzzy graph if $\mu(x, y) = \min(\sigma(x), \sigma(y))$ for all x, $y \varepsilon V$

An edge e = xy of a fuzzy graph is called an effective edge if $\mu(x,y) = \min(\sigma(x),\sigma(y))$

$N(x) = \{ y \ V / \mu(x,y) = \sigma(x) \wedge \sigma(y)\}$ is called the neighbourhood of x and $N[x] = N(x) \cup \{x\}$ is the closed neighbourhood of x. Let $G = (\sigma, \mu)$ be a fuzzy graph on V. Let x, y, ε V. We say that x dominates y in G if $\mu(x,y) = \min(\sigma(x), \sigma(y))$

A vertex u of a fuzzy graph is said to be an isolated vertex if $\mu(u,v) < \sigma(u) \wedge \sigma(v)$ for all $v \varepsilon V / \{u\}$ That $N(u) = \psi$

A set $D \subseteq V$ is a dominating set of a fuzzy graph $G = (\sigma, \mu)$ if every vertex in V-D is effective adjacent to some vertex in D. The smallest number of vertices in any dominating set of G is called its fuzzy domination number and is denoted by $\gamma(G)$ or γ. A dominating set is called minimum if it contains γ elements.

A path P in a fuzzy graph $G = (\sigma, \mu)$ is a sequence of distinct vertices $x_0, x_1 \ldots\ldots\ldots\ldots x_n$ such that $\mu(x_{i-1}, x_i) > 0, 1 \leq i \leq n$. Here $n \geq 1$ is called the length of a path. A path P is called a cycle if $x_0 = x_n$ and $n \geq 3$. A fuzzy tree is an acyclic and connected fuzzy graph.

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Two vertices are said to be effective adjacent if they are the end vertices of the same effective edge.

Let G = (σ, μ) be a fuzzy graph on V. Then the effective incident degree of a fuzzy graph is defined as number of effective incident edges on a vertex v_i. It is denoted as $d_{E1}(v_i)$.

The minimum effective incident degree of a fuzzy graph G is defined by $\delta_{E1}(G) = \wedge \{ d_{E1}(G) : v \varepsilon V\}$

The maximum effective incident degree of a fuzzy graph G is defined by

$\Delta_{E1}(G) = \vee \{d_{E1}(G) : v \varepsilon V\}$

A path P is called effective path if each edge in a path P is an effective edge.

An effective path P is called an effective cycle if $x_0 = x_n$ and $n \geq 3$.

A fuzzy graph G = (σ, μ) is said to be effective connected if there exists an effective path between every pair of vertices.

A fuzzy graph G = (V, σ, μ) is said to be complete if each vertex is of effective incident degree (n-1) .

In an effective connected fuzzy graph G = (σ , μ) the effective distance d (v_i, v_j) between two of its vertices v_i and v_j is the length of the shortest effective path between them.

2. Fuzzy Effective Spanning Tree:

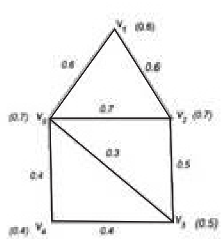


Figure 2.1

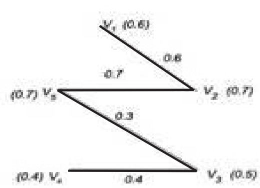


Figure 2.2

Defintion 2.1 A Fuzzy Effective tree is an effective connected fuzzy graph without any of effective cycle.

Defintion 2.2 A fuzzy graph G is said to be fuzzy effective connected if there is at least one effective path between every pair of vertices in an effective fuzzy graph G.

Definition 2.3

The effective tree T is said to be a fuzzy effective spanning tree of a fuzzy effective connected graph G if T is an effective sub graph of an effective fuzzy graph G and T contains all vertices of G.

Definition 2.4

An effective edge in an effective spanning tree T of an effective fuzzy graph G is called a fuzzy effective branch of T.

Definition 2.5

An effective edge of an effective fuzzy graph G that is not in a given fuzzy effective spanning tree T is called a fuzzy effective chord.

Definition 2.6

The fuzzy effective rank of an effective fuzzy graph G is the number of effective branches in any fuzzy effective spanning tree of G.

Definition 2.7

The fuzzy effective Nullity of a effective fuzzy graph G is the number of effective chords in G.

Definition 2.8

The fuzzy effective spanning tree with the smallest weight in a weighted effective fuzzy graph is called a shortest fuzzy effective spanning tree or shortest distance effective fuzzy spanning tree or minimal effective fuzzy spanning tree.

Definition 2.9

The fuzzy effective distance between two fuzzy effective spanning tree T_i and T_j of an effective fuzzy graph G is defined as the number of effective edges of G present in one tree but not in the other.

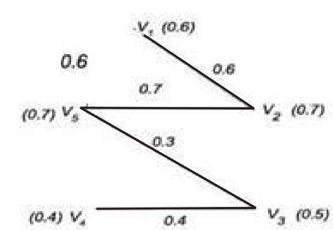


Figure 2.3

Definition 2.10

The fuzzy effective ring sum $T_i \oplus T_j$ is the subgraph of effective fuzzy graph G containing all effective edges of G that are either in T_i or in T_j but not in both.

Definition 2.11

An effective circuit formed by adding an effective chord to a fuzzy effective spanning tree is called a fuzzy effective fundamental circuit.

Definition:2.12

The generation of one fuzzy effective spanning tree from another, through addition of an effective chord and deletion of an appropriate effective branch is called a fuzzy effective cyclic interchange.

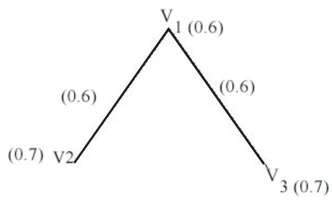


Figure 2.4

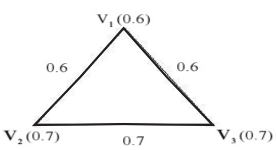


Figure 2.5

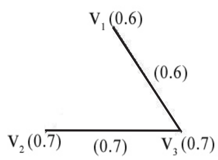


Figure 2.6

Theorem 2.1

Every effective connected fuzzy graph has at least one fuzzy effective spanning tree.

Proof : case 1

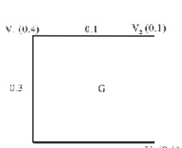
Suppose G has no effective circuit, it is its own fuzzy effective spanning tree.

Case 2

If G has a fuzzy effective circuit, delete an effective edge from the fuzzy effective circuit. This will still leave the fuzzy graph effective connected. If there are more fuzzy effective circuit, repeat the operation till an effective edge from the last effective circuit is deleted leaving an effective connected, effective circuit free effective fuzzy graph that contains all the vertices of G. Therefore, Every effective connected graph has at least one fuzzy effective spanning tree.

Theorem 2.2

An effective connected fuzzy graph G is a fuzzy effective tree if and only if adding an effective edge between any two vertices in G creates exactly one fuzzy effective circuit.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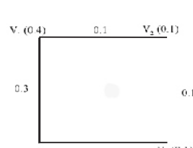
Figure 2.8

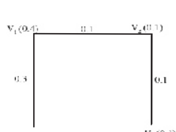
Figure 2.9

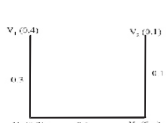
Figure 2.10

Proof

An effective connected fuzzy graph G is a fuzzy effective tree, there exists one and only effective path between every pair of vertices in an effective tree G, Now adding an effective edge between them creates an additional effective path. The union of these two effective paths creates exactly one fuzzy effective circuit.

Converesely,Suppose adding an effective edge between any two vertices in G creates exactly one fuzzy effective circuit, Now remove an edge between any two vertices of G gives exactly one effective path. If in an effective fuzzy graph G there is one and only one effective path between every pair of vertices, therefore G is a fuzzy effective tree.

Theorem 2.3

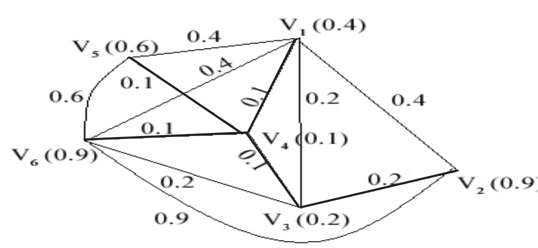

Figure 2.11

A fuzzy effective Spanning tree T is a shortest fuzzy effective spanning tree, if and only if there exists no other fuzzy effective spanning tree of effective fuzzy graph G at a effective distance of one from T whose weight is smaller than that of T.

Proof

Suppose T be a shortest Fuzzy Effective Spanning tree, from the definition of shortest fuzzy effective spanning tree, no other fuzzy effective spanning tree has weight smaller than T. This part is trivial

Conversely Let T_1 be a Fuzzy Effective spanning tree in G satisfying the hypothesis of the theorem. That is there is no fuzzy effective spanning tree at a effective distance of one from T_1 which is shorter than T_1 Now to prove if T_2 is a fuzzy effective shortest spanning tree in G, the weight of T_1 will also be equal to that of T_2. Let T_2 be a shortest fuzzy effective spanning tree in G. clearly T_2 must also satisfy the hypothesis of the theorem.

Consider an effective edge e in T_2 which is not in T_1. Adding e to T_1 forms a fuzzy effective fundamental circuit with fuzzy effective branches in T_1. Some, but not all of the effective branches in T_1 that from the fuzzy effective fundamental circuit with e may also be in T_2, each of these effective branches in T_1 has a weight smaller than or equal to that of e, because of the assumption on T_1. Amongst all those effective edges in this fuzzy effective circuit which are not in T_2 at least one say b_j, must form some fuzzy effective fundamental circuit containing e, Because of the minimality assumption on T_2 weight of b_j cannot be less than that of e. Therefore b_j must have the same weight as e. Hence the fuzzy effective spanning tree $T_1 = (T_1 \text{ U } e\text{-}b_j)$ obtained from T_1 through one effective cycle exchange has the same weight as T_1 But T_1 has one effective edge more in common with T_2, and it satisfies the condition of theorem. This argument can be repeated producing a series of effective trees of equal weight, T_1, T_1, T_1 …………… Each a unit effective distance closer to T_2, until we get T_2 itself This proves that if none of the effective spanning trees at a unit distance from T is shorter than T, no fuzzy effective spanning tree shorter than T exists in the effective fuzzy graph.

Algorithm for fuzzy effective shortest spanning tree 2.1

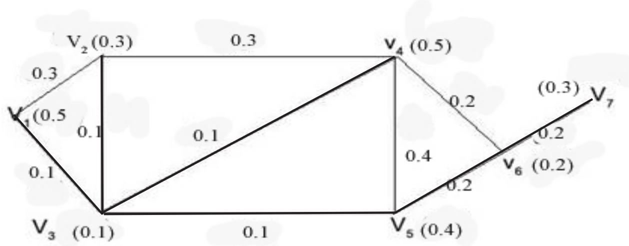

Figure 2.12

	V_1	V_2	V_3	V_4	V_5	V_6	V_7
V_1	0	0.3	0.1	0	0	0	0
V_2	0.3	0	0.1	0.3	0	0	0
V_3	0.1	0.1	0	0.1	0.1	0	0
V_4	0	0.3	0.1	0	0.4	0.2	0
V_5	0	0	0.1	0.4	0	0.2	0
V6	0	0	0	0.2	0.2	0	0.2
V_7	0	0	0	0	0	0.2	0

Step 1. List all effective edges of the effective fuzzy graph G in order of non decreasing weight.

Step 2. Select a smallest effective edge of G.

Step 3. From all the remaining effective edges of G select another smallest effective edge that makes no

fuzzy Effective circuit with the previously selected effective edges.

Step 4. Continue until n-1 effective edges have been selected, and these effective edges will constitute the Desired fuzzy effective shortest spanning tree.

Algorithm – Matrix Representation 2.2

Step1. Select n isolated vertices and label them V_1, V_2 ………………..V_n

Step 2. Tabulate the given weights of the effective edges of fuzzy graph G in an n by n matrix.

Step 3. The entries in the table are symmetric with respect to the diagonal, and the diagonal is zero.

Step 4. Set the weights of non-existent effective edges are denoting zero and weights of self loops also zero.

Step 5. Start from vertex V_1 and connect it to its nearest neighbour that is to the vertex which has the

Smallest entry in row 1 of the table say V_k

Step 6. Consider V_1 and V_k as one effective fuzzy subgraph and connect this effective fuzzy subgraph to its closest neighbour that is, to a vertex other than V_1 and V_k that has the smallest entry among all entries in rows 1 and K. Let this new vertex be V_i.

Step 7. Regard the effective tree with vertices V1, Vk and Vi as one effective fuzzy subgraph and continue the process until all n vertices have been onnected by n-1 fuzzy effective eges.

Conclusion:

It has been shown that the fuzzy effective shortest spanning tree algorithm is useful for designing fuzzy relational data base. This will lead us to a new notion for fuzzy graph. From a practical point of view the fuzzy effective shortest spanning tree may be built from many different kinds of functions.

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