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Numerical solution of magneto-hydrodynamic and Hematocrit on blood flow

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Abstract

Fluid flow analysis of blood flow through a tapered artery with mild stenosis in the presence of magnetic field is investigated. In this paper the effect of magnetic field and shape of stenosis on the flow rate and pressure gradient is studied. The blood flowing through the artery is considered to be Newtonian. This model is consistent with the principles of Ferro hydrodynamics and magneto hydrodynamics (MHD). The results indicate that rise in systolic pressure and fall in diastolic pressure are very harmful for weak heart.

Keywords: blood flow, MHD, Newtonian fluid, tapered artery, stenosis

INTRODUCTION

Investigation of the blood flow in a stenosed (constricted) geometry is of interest because of its significance in relation to human vascular diseases. The study of blood flow through constricted arteries plays a significant role in the analysis of the effects of constrictions on flow that might have severe consequences because the flow of blood gets disturbed up to the limit based on the severity of the constrictions developed in the arteries. Stenosis has great effect on blood flow through and beyond the narrowed arterial segment. Atherosclerotic disease occurs in the regions of irregularity such as curved and tapered narteries and stenotic sites. These irregularities are believed to be partially responsible for the appearance of atherosclerotic that affects the function of the cardiovascular system (Lee and Fung, 1970; Fry, 1973; Young, 1979; Asakura and Karino, 1990). The red blood cell (RBC) is a major biomagnetic substance, and the blood flow may be influenced by the magnetic field. The decrease in blood flow was caused by either an increase in blood resistance or a decrease in blood pressure (Kuchel and Bulliman, 1989; Schenck *et al.*, 1992).The properties of human blood as well as blood vessels and magnetic field effect were the subjects of interest for several researchers (Pauling and Coryell, 1936; Keltner *et al.*, 1990; Higashi *et al.*, 1993; Ichioka *et.al.*,1998;2000).Based on these views, in this paper an attempt has been done to find the effects of the Hematocrit and magnetic field on the blood flow through the artery with multiple mild stenoses, by considering the fact that the viscosity and Hematocrit vary with the radius of the artery. The expressions for the flow rate and pressure gradient for the Newtonian fluid are found numerically and results are discussed with respect to Hematocrit and Hartmann number by showing the graphs.

ASSUMPTIONS

1. Blood is assumed to be Newtonian, incompressible and homogeneous fluid.
2. Blood flowing is a suspension of RBC in plasma.
3. Viscosity of blood varies radially.
4. Density is constant.
5. Magnetic field applied externally is of constant strength.
6. The electromagnetic force produced is very small.
7. Electrical conductivity is small.
8. Cylindrical polar co-ordinates are used.
9. The axis of symmetry of the artery taken as z-axis.
10. The stenosis develops symmetrically about the axis of the artery but not symmetric radial co-ordinates.
11. Stenosis is tapered.(Mild Stenosis).
12. Tapered angle is very very small.

NOTATIONS

(r,θ,z) : Cylindrical polar co-ordinate system

w : Axial velocity component

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p : Pressure of the fluid (blood)

σ: Electrical conductivity

β_0 : Electromagnetic induction

μ_p : Viscosity of plasma

$\mu (r)$: Viscosity the fluid (blood)

$h(r)$: Hematocrit

H : Maximum Hematocrit

R_1 : Radius of normal artery

R (z) : Effective radius of artery

Φ : angle of tapering

H : h cos φ= height of stenosis in tapered artery

L_0 : The length of the stenosis

m: tanφ = slope of tapered vessel

THE MATHEMATICAL MODEL

By assuming the segment of artery is as rigid cylindrical tube in the presence of tapered artery with mild stenosis and magnetic field, cylindrical system (r, θ, z) is considered. It is assumed that there is no flow of blood in θ direction in the artery, so system is reduced to (r,z) co-ordinates. Further Newtonian, laminar and steady flow of blood is taken in the artery. Thus, one dimensional equation of motion for the steady and axially symmetric flow of blood through an artery provided with a mild stenosis under the above mentioned assumption is (Verma and Parihar, 2010)

$$\frac{dp}{dz} + \frac{1}{r}\frac{\delta}{\delta r}\left(-r\mu(r)\frac{dw}{dr}\right) + \beta_0^2\sigma w = 0 \quad \text{................(1)}$$

The relation between blood viscosity, μ(r) and Hematocrit is given by (Einstein, 1906)

$$\mu(r) = \mu_p\left[1 + 2.5h(r)\right] \quad \text{..............(2)}$$

where, μ_p is coefficient of viscosity of plasma and h(r) is Hematocrit.

The relation between Hematocrit and maximum Hematocrit (Lih, 1975) is given by

$$h(r) = H\left[1 - \left(\frac{r}{R_0}\right)^n\right] \quad \text{..................(3)}$$

where H is maximum Hematocrit at the center of the tube, R_0 is radius of normal tube (without stenosis) and $n \geq 2$ is a parameter determining the shape of Hematocrit profile. This shape of the profile given by equation (3) is valid only for very dilute suspension of red cells which are supposed to be spherical in shape.

The boundary conditions are given as

$$\frac{\partial w}{\partial r} = 0 ; r = 0$$
$$w = 0 ; r = R(z) \quad \text{.......................(4)}$$

This gives the Maximum velocity at the centerline and there is no-slip velocity at the wall.

Guo-Tao *et al.*, (2004) discussed the physical model of tapered artery with stenosis. In such a case the geometry of the artery is modeled mathematically as follows:

$$\frac{R(z)}{R_1} = \begin{cases} 1 - \frac{m(z+L)}{R_1} ; 0 \leq z \leq d_0 \\ 1 - \frac{m(z+L)}{R_1} - \frac{h\cos\phi}{2R_1}\left[1 + \cos\left(\frac{\pi z}{L_0}\right)\right] ; d_0 \leq z \leq L_0 + d_0 \\ 1 - \frac{m(z+L)}{R_1} ; L_0 + d_0 \leq z \leq L \end{cases} \quad \text{...............(5)}$$

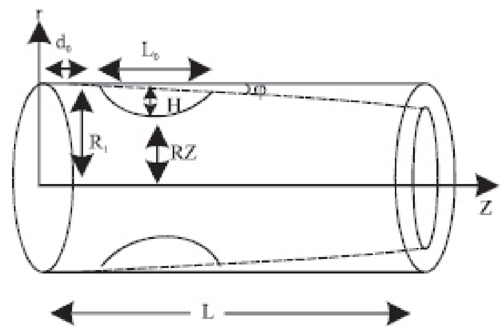

Figure 1. Physical model and co-ordinate system

ANALYSIS OF MATHEMATICAL MODEL

To analyze the mathematical model, in a simple and easy manner, linear transformation is used to find the solution of second order partial differential equation which is given by equation (1).

The linear transformation is taken by

$$x = \frac{r}{R_1} \quad \text{.................. (6)}$$

By using equations (2), (3) and (6) in to equation (1) is transformed in to in ordinary differential equation by

$$\frac{d}{dx}\left[x\left(a_1 - a_2x^n\right)\frac{dw}{dx}\right] - M^2xw = \frac{R_1^2}{\mu_p}\frac{dp}{dz}x \quad \text{........................ (7)}$$

where,

$$a_1 = 1 + a_2 \quad \text{........................(8)}$$

$$a_2 = 2.5\text{H} \quad \text{.......................(9)}$$

$$M = \beta_0 R_1 \left(\frac{\sigma}{\mu_p,} \right)^{\frac{1}{2}}$$ Hartmann Number …….(10)

By using equation (6), the boundary conditions(4) are given by

$$w = 0, x = \frac{R(z)}{R_1}$$

$$\frac{dw}{dx} = 0, x = 0 \qquad \text{………………….(11)}$$

SOLUTION BY QUASI POWER SERIES METHOD

Equation (7) is second order differential equation with variation coefficient is solved by Quasi Power Series method. According to the problem, it is need to find the solution near at x=0.

The series solution of equation (7) is given by

$$w = T\sum_{i=0}^{\infty} S_i x^i + \frac{R_1^2}{4a_1\mu_p}\frac{dp}{dz}\sum_{i=0}^{\infty} s_i x^{i+2} \qquad \text{………………..(12)}$$

where, second term of the right side of equation (12), is the solution corresponding to non homogenous part of equation (7) and s_i's are series constant.

Assume that, $S_0 = 1$, and by calculation

$S_1 = 0, S_2 = M^2/4a_1$ and

$$S_{i+j} = \frac{a_2(i)(i+j)S_i + M^2 S_{i+j-2}}{a_1(i+j)^2}; i \geq 0 \qquad \text{………..(13)}$$

taking j = 3, we get,

$$S_{i+3} = \frac{a_2(i)(i+3)S_i + M^2 S_{i+1}}{a_1(i+3)^2}$$

… ....(13a)

Further putting i = 0,1,2,3… respectively in eqn.(13 a), we get,

$S_3=0, S_4= M^4/64a\text{-}_1^2,\ldots$

Similarly,

Assume that s_0=1and by calculation $s_1=0, s_2= M^2/16a_1$ and

$$s_{i+j} = \frac{a_2(i+2)(i+j+2)s_i + M^2 s_{i+j-2}}{a_1(i+j+2)^2}; i \geq= 0 \qquad \text{………(14)}$$

taking j=3 and i=0, 1, 2, 3…, in eqn. (14), we get,

$s_3 = 2a_2/5a_1, s_4 = M^4/576\, a_1^2, \ldots$

Using boundary conditions(11), using this equation (12), T is determined,

$$T = \frac{-\dfrac{R_1^2 \frac{dp}{dz}}{4a_1\mu_p}\displaystyle\sum_{i=0}^{\infty} s_i \left(\frac{R(z)}{R_1}\right)^{i+2}}{\displaystyle\sum_{i=0}^{\infty} S_i \left(\frac{R(z)}{R_1}\right)^{i}} \qquad \text{……………(15)}$$

The velocity profile, w is given by

$$w = \frac{-\left(\dfrac{R_1^2 \frac{dp}{dz}}{4a_1\mu_p}\right)(AB - CD)}{D} \qquad \text{....…………(16)}$$

where,

$$A = \left\{ s_0\left(\frac{R}{R_1}\right)^2 + s_1\left(\frac{R}{R_1}\right)^3 + s_2\left(\frac{R}{R_1}\right)^4 + s_3\left(\frac{R}{R_1}\right)^5 + \ldots \right\}$$

$$B = \left\{ S_0 + S_1 x + S_2 x^2 + \ldots \right\}$$

$$C = \left\{ s_0 x^2 + s_1 x^3 + s_2 x^4 + \ldots \right\}$$

$$D = S_0 + S_1\left(\frac{R}{R_1}\right)^1 + S_2\left(\frac{R}{R_1}\right)^2 + S_3\left(\frac{R}{R_1}\right)^3 + \ldots$$

The volumetric flow Q of fluid in stenotic region given by

$$Q = \int_0^{\frac{R}{R_1}} 2\pi R_1 x\, w(x)\, dx \qquad \text{……..(17)}$$

Let Q_p denotes the flow of plasma fluid in unconstructed tube when M=H=0. Then

$$Q_p = -\pi \left(\frac{R_1^3}{8\mu_p}\right)\left(\frac{dp}{dz}\right)_0 \qquad \text{…………..(18)}$$

where, $\left(\frac{dp}{dz}\right)_0$ denotes pressure gradient of flow in the normal tube in the absence of magnetic field and Hematocrit.

The non-dimensional flow $Q_L = Q/Q_p$

$$Q_L = \frac{Q}{Q_p} = \frac{G\left(\frac{R}{R_1}\right)^4}{G_0\left(a_1 + \frac{M^2}{4}\left(\frac{R}{R_1}\right)^2\right)} \qquad \text{……………..(19)}$$

$$G = \frac{dp}{dz} \text{ and } G_0 = \left(\frac{dp}{dz}\right)_0$$

The non-dimensional pressure gradient, $G_L = G/G_0$

$$G_L = Q_L \left[\frac{\frac{a_1}{4} + \frac{M^2}{16}\left(\frac{R}{R_1}\right)^2}{\frac{1}{4}\left(\frac{R}{R_1}\right)^4} \right] \quad \text{.............................(20)}$$

RESULTS AND DISCUSSION

Expression for the flow rate and pressure gradient are obtained for different values of magnetic parameter (M) and Hematocrit parameter (H) Results of calculations for Newtonian fluid for these expressions are shown by curves in respective figures. The expression of the non-dimensional flow rate is given by equation (19) with respect to the length of artery; Hematocrit (H) and

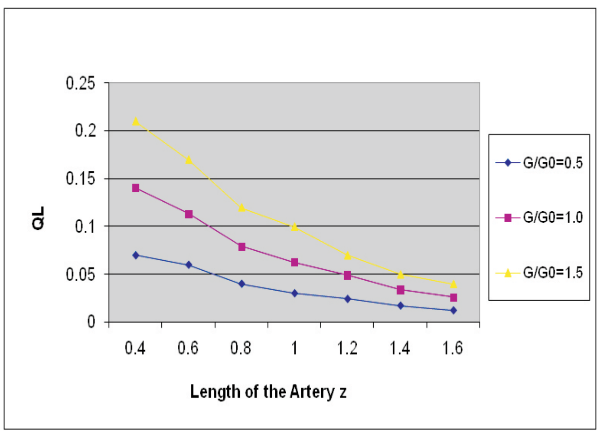

Figure 1

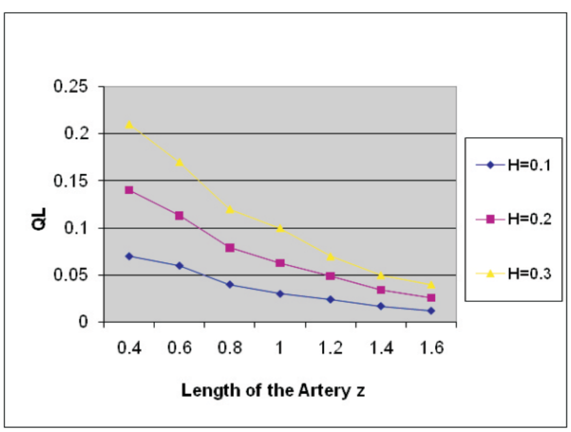

Figure 2

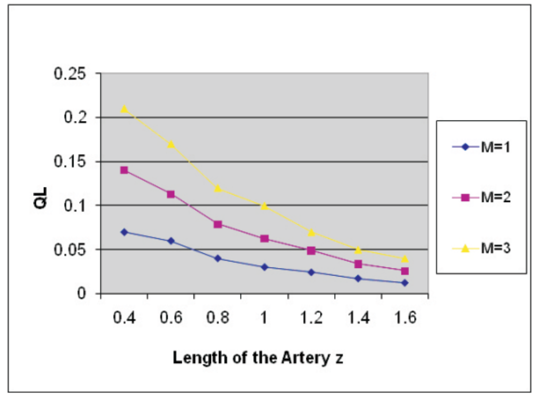

Figure 3

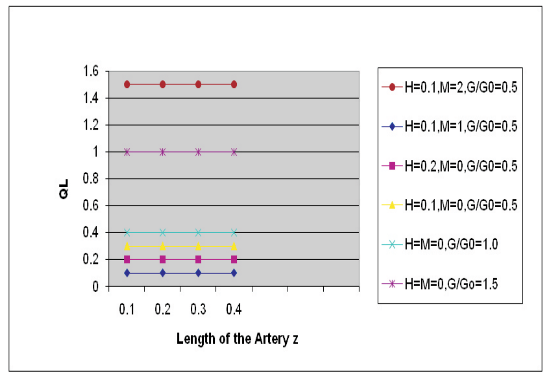

Figure 4

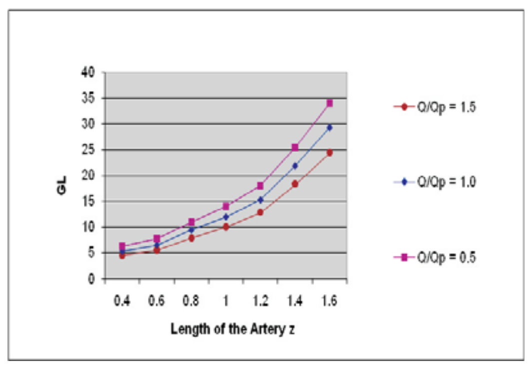

Figure 5

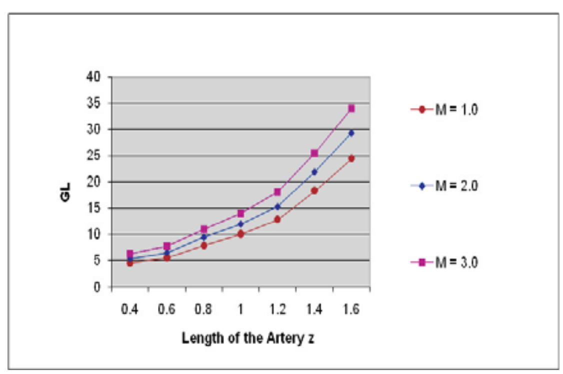

Figure 6

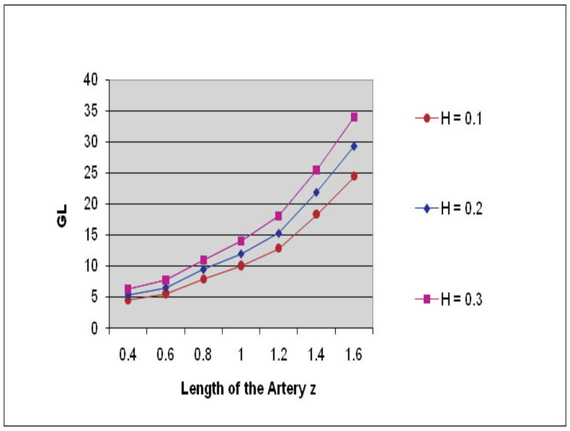

Figure 7

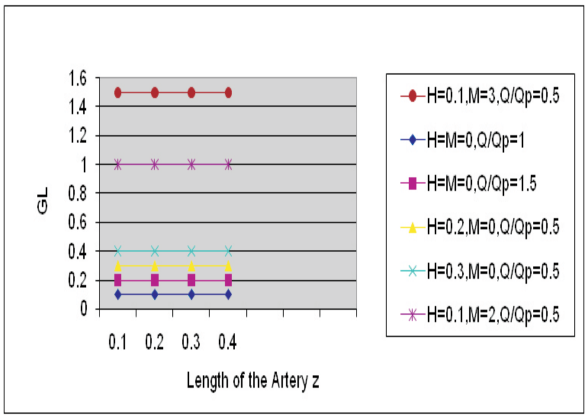

Figure 8

Hartmann number (M) are plotted in Graphs 1,2,3,and 4 respectively.

In the presence of stenosis, as it is seen from Figure 1, figures as the value of pressure gradient increases, Q_L (n.d. Q) also increases while other parameters are fixed. Similarly it is seen from

Figures 2 and 3, as the value of M and H increase the Q_L (n.d. Q) decreases, while other parameters are fixed, when stenosis are taken. Similarly, in the absence of stenosis, Figure 4, depicts that as the value of pressure gradient increases, Q_L (n.d. Q) also increases, while other

parameters are fixed but as the value of M and H increase the Q_L (n.d. Q) decreases while other parameters are fixed.

Similarly, in the presence of stenosis, it is seen from Figure 5, that as the value of flow rate

increases, non-dimensional pressure gradient (n.d.G), G_L also increases, while other parameters are fixed. Similarly it is seen from Figures 6 and 7, that as the value of M and H increase, G_L (n.d.G) also increases while other parameters are fixed, when stenosis are taken.

Similarly, in the absence of stenosis, Figure 8 depicts that as the value of M and H increase, G_L (n.d. Q) decreases while other parameters are fixed.

CONCLUSION

Our study revealed that for a particular value of length of artery (z), the non-dimensional pressure gradient increase for both M and H increase, while nondimensional flow rate decreases. This type of increase in M and H indicated that the rise in systolic pressure and fall in diastolic pressure are very harmful for weak heart. The rise for M is much more important than that for H. Therefore, it can be concluded through our mathematical findings that the magneticity is more destructive for diseased cardiovascular system.

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