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On Fuzzy polynomial equations

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Abstract

In this article, a new ranking method with the aid LR type fuzzy numbers, to find the real roots for fuzzy polynomial equations is proposed. We transform fuzzy polynomial equation to system of crisp polynomial equations, this transformation performs with ranking method based on three parameters such as Real number value, Vagueness and Fuzziness. Some numerical examples to illustrate our ranking method are also included.

Keywords: fuzziness, fuzzy polynomial equation, LR type fuzzy numbers, real number value, vagueness

INTRODUCTION

Polynomial equations play an important role in various areas such as mathematics, engineering and social sciences. It is quiet interesting to solve the polynomial equations in fuzzy environment. For the fuzzy polynomial equation, we mean all the coefficients A's are fuzzy numbers. This ranking method is provided for canonical representation of the solution of fuzzy linear system by Nehi *et al.*, (2006). The applications of fuzzy polynomial equations are considered by Abbasbandy and Amirfakhrian (2006). Also, numerical solution of fuzzy polynomial equations by fuzzy neural network is solved by Abbasbandy and Otadi (2006) and linear and non-linear fuzzy equations are shown by Delgado *et al.*, (1998a); Abbasbandy and Asady (2004) and Abbasbandy and Alavi (2005).

In this paper, we introduce a new method for solving a fuzzy polynomial equation based on ranking method which is introduced by Delgado *et al.*, (1998a,b). They introduced three real indices called value, ambiguity and fuzziness to obtain "Simple", fuzzy numbers, that could be used to represent more arbitrary fuzzy numbers. Rouhparvar (2007) used the same parameters and transformed fuzzy polynomial equation to system of crisp polynomial equations. In this work, we transform fuzzy polynomial equation to system of crisp polynomial equations using three parameters such as real number value, vagueness and fuzziness.

In section 2, some basic definitions and the notions of fuzzy numbers and fuzziness are given. In section 3, we present fuzzy polynomial equation and proposed method for solving it and provide two illustrative examples. Some numerical examples are considered in section 4 and conclusion comes in section 5.

Basics

In this section, some preliminaries which are needed for our work are presented.

2.1.Definition

A fuzzy subset A of the real line R with membership function A(x): $\Re \longrightarrow [0,1]$ is called a fuzzy number if

(i) A is normal, ie., there exist an element x_0 such that $A(x_0) = 1$,

(ii) A is fuzzy convex, ie.,

$$A(\lambda x_1 + (1-\lambda) x_2) \geq \min\{A(x_1), A(x_2)\} \ \forall x_1, x_2 \in \Re, \ \forall \lambda \in [0,1]$$

(iii) A (x) is upper semi continuous,

(iv) Support A is bounded, where Support $A = cl\{x \in \Re : A(x) > 0\}$,

and cl is the closure operator.

2.2. Defintion

A fuzzy set A on $\Re$ is a fuzzy number if

i) Its membership function is upper semi continuous.

ii) There exist three intervals

[a,b], [b,c], [c,d], such that A is increasing on [a,b], equal to 1 on [b,c], decreasing on [c,d], and equal to 0 elsewhere.

2.3. Remark

In this article, we introduce the following parameters as R(A) the real number value of A, V(A) the vagueness of A and F(A) the fuzziness of A.

Also referring from (Delgado *et al.*, 1998b; Ma *et al.*, 1999), we have for trapezoidal fuzzy numbers.

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$$R(A_i) = \frac{a_i + b_i}{2} + \frac{\beta_i - \alpha_i}{3} \quad (1)$$

$$V(A_i) = \frac{b_i - a_i}{2} + \frac{\beta_i + \alpha_i}{3} \quad (2)$$

$$F(A_i) = \frac{\beta_i + \alpha_i}{2} \quad (3)$$

Also for triangular fuzzy numbers

$$R(A_i) = m_i + \left(\frac{\beta_i - \alpha_i}{3}\right) \quad (4)$$

$$V(A_i) = \frac{\beta_i + \alpha_i}{3} \quad (5)$$

$$F(A_i) = \frac{\beta_i + \alpha_i}{2} \quad (6)$$

R(A) may be seen as a central value which represents the real number value of a fuzzy number, V(A) the global spread of the membership function of A, the measure of the vagueness of A and F(A) the global difference between A and A^c the complement of A.

Note that for any two trapezoidal fuzzy number A_1 and A_2 and any real number k we have,

$$R(k A_1 + A_2) = kR(A_1) + R(A_2) \quad (7)$$

$$V(k A_1 + A_2) = |k| V(A_1) + V(A_2) \quad (8)$$

$$F(kA_1 + A_2) = |k| F(A_1) + F(A_2) \quad (9)$$

2.4. Note

We say that $A_1 = A_2$ if and only if :

$$\left.\begin{array}{l} R(A_1) = R(A_2) \\ V(A_1) = V(A_2) \\ F(A_1) = F(A_2) \end{array}\right\} \quad (10)$$

3. The fuzzy polynomial equation

In this section, we propose a procedure for solving fuzzy polynomial equation, then review two illustrative examples of the trapezoidal fuzzy number and the triangular fuzzy number.

3.1. Definition

The polynomial equation

$$A_1 x + A_2 x^2 + ... + A_n x^n = A_0 \quad (11)$$

Where $x \in R$, the co-efficient $A_1, A_2, ... A_n$ and A_0 are fuzzy numbers is called a fuzzy polynomial equation.

Now let x be a real solution for (11), then by (10) we have,

$$R(A_1 x + A_2 x^2 + ... + A_n x^n) = R(A_0)$$

$$V(A_1 x + A_2 x^2 + ... + A_n x^n) = V(A_0)$$

$$F(A_1 x + A_2 x^2 + ... + A_n x^n) = F(A_0)$$

of (7), (8) and (9) implies

$$\left.\begin{array}{l} R(A_1)x + ...+ R(A_n)x^n = R(A_0) \\ V(A_1)|x| + ...+ V(A_n)|x^n| = V(A_0) \\ F(A_1)|x| + ...+ F(A_n)|x^n| = F(A_0) \end{array}\right\} \quad (12)$$

Hence for finding the real solutions of (11), it is enough to solve the above system of crisp polynomial equations with crisp methods. Of course, though we have a system of crisp polynomial equations but it is only enough to solve one equation of (12) and finding its real roots, then real roots of that equation satisfy in both other equations are the real solutions of system (12). Also for solving (12) one can consider two state: the first, we suppose $x \geq 0$ then have

$$\left.\begin{array}{l} R(A_1)x + ...+ R(A_n)x^n = R(A_0) \\ V(A_1)x + ...+ V(A_n)x^n = V(A_0) \\ F(A_1)x + ...+ F(A_n)x^n = F(A_0) \end{array}\right\} \quad (13)$$

then we obtain positive real roots. The second, we suppose $x < 0$ then have

$$\left.\begin{array}{l} R(A_1)x + ...+ R(A_n)x^n = R(A_0) \\ -V(A_1)x + ... + (-1)^n V(A_n)x^n = V(A_0) \\ -F(A_1)x + ... + (-1)^n F(A_n)x^n = F(A_0) \end{array}\right\} \quad (14)$$

where we obtain negative real roots.

3.2. Example

Let $A_i = (a_i, b_i, \alpha_i, \beta_i)$ (i = 0, 1, ..., n) trapezoidal fuzzy number then by (1), (2) and (3) we have

$$\left(\frac{a_1+b_1}{2}+\frac{\beta_1-\alpha_1}{3}\right)x + ... + \left(\frac{a_n+b_n}{2}+\frac{\beta_n-\alpha_n}{3}\right)x^n = \left(\frac{a_0+b_0}{2}+\frac{\beta_0-\alpha_0}{3}\right)$$

$$\left(\frac{b_1-a_1}{2}+\frac{\beta_1+\alpha_1}{3}\right)|x| + ... + \left(\frac{b_n-a_n}{2}+\frac{\beta_n+\alpha_n}{3}\right)|x^n| = \left(\frac{b_0-a_0}{2}+\frac{\beta_0+\alpha_0}{3}\right)$$

$$\left(\frac{\beta_1+\alpha_1}{2}\right)|x| + ... + \left(\frac{\beta_n+\alpha_n}{2}\right)|x^n| = \left(\frac{\beta_0+\alpha_0}{2}\right) \quad (15)$$

Thus the system (12) can be solved to get real roots by using (15).

3.3 Example

In this example we see that for any triangular fuzzy number. $A_i=(m_i,\alpha_i,\beta_i)$, (i=0, 1, ..., n) fuzzy polynomial equation (11) is transformed to system of crisp polynomial equations.

We have from (4), (5) and (6)

$$\left(m_1+\frac{\beta_1-\alpha_1}{3}\right)x+\dots\dots+\left(m_n+\frac{\beta_n-\alpha_n}{3}\right)x^n=\left(m_0+\frac{\beta_0-\alpha_0}{3}\right)$$
$$\left(\frac{\beta_1+\alpha_1}{3}\right)|x|+\dots..+\left(\frac{\beta_n+\alpha_n}{3}\right)|x^n|=\left(\frac{\beta_0+\alpha_0}{3}\right)$$
$$\left(\frac{\beta_1+\alpha_1}{2}\right)|x|+\dots..+\left(\frac{\beta_n+\alpha_n}{2}\right)|x^n|=\left(\frac{\beta_0+\alpha_0}{2}\right) \quad (16)$$

We solve problem (16) according to solving method of system (12).

4. Numerical Applications

4.1. Application

Considering ,

$(5, 3, 4)\ x + (4, 2, 3)x^2 = (9, 5, 7)$

The above fuzzy polynomial equation can be solved as:

By (4,5,6), we have

$R(A_1)=\frac{16}{3}$, $R(A_2)=\frac{13}{3}$, $R(A_0)=\frac{29}{3}$

$V(A_1)=\frac{7}{3}$, $V(A_2)=\frac{5}{3}$, $V(A_0)=\frac{12}{3}$

$F(A_1)=\frac{7}{2}$, $F(A_2)=\frac{5}{2}$, $F(A_0)=\frac{12}{2}$

By (12) we have

$$\left.\begin{aligned}\frac{16}{3}x+\frac{13}{3}x^2&=\frac{29}{3}\\ \frac{7}{3}|x|+\frac{5}{3}|x^2|&=\frac{12}{3}\\ \frac{7}{2}|x|+\frac{5}{2}|x^2|&=\frac{12}{2}\end{aligned}\right\} \quad (17)$$

By solving (17), we have the exact solution as x = 1.

4.2. Application

Considering,

$(0,1,1)\ x + (0,2,2)\ x^2 + (1,1,1)\ x^3 = (-1, 4, 4)$

The above fuzzy polynomial equation can be solved as:

By (4,5,6), we have

$R(A_1) = 0$, $R(A_2) = 0$, $R(A_3) = 1$

$V(A_1)=\frac{2}{3}$, $V(A_2)=\frac{4}{3}$, $V(A_3)=\frac{2}{3}$

$F(A_1) = 1$, $F(A_2) = 2$, $F(A_3) = 1$

$R(A_0) = 1$, $V(A_0)=\frac{8}{3}$, $F(A_0) = 4$

By (12) we have

$$\left.\begin{aligned}x^3&=-1\\ |x|+2|x^2|+|x^3|&=4\\ |x|+2|x^2|+|x^3|&=4\end{aligned}\right\} \quad (18)$$

By solving system (18), we have the exact solution as x = -1.

4.3. Application

Considering,

$(0, 1, 1)\ x + (0, 1, 1)x^2 = (14, 2, 5).$

The above fuzzy polynomial equation can be solved as:

By (4,5,6), we have

$R(A_1) = 0$, $R(A_2) = 0$, $R(A_0) = 15$

$V(A_1)=\frac{2}{3}$, $V(A_2)=\frac{2}{3}$, $V(A_0)=\frac{7}{3}$

$F(A_1) = 1$, $F(A_2) =1$, $F(A_0)=\frac{7}{2}$

By (12) we have

$$\left.\begin{aligned}0&=15\\ \frac{2}{3}|x|\frac{2}{3}|x^2|&=\frac{7}{3}\\ |x|+|x^2|&=\frac{7}{2}\end{aligned}\right\} \quad (19)$$

Note that, system (19) is inconsistent and has no real solution.

4.4. Application

Considering,

$(2,4,1,2)\ x + (0,1,3,1)\ x^2+(1,1,½,½)x^3 = (-5,0,11/2,5/2)$

The above fuzzy polynomial equation can be solved as:

By (1) (2) and (3), we have

$R(A_1)=\frac{10}{3}$, $R(A_2)=\frac{1}{6}$, $R(A_3) = 0$, $R(A_0)=\frac{7}{2}$

$V(A_1) = 2$, $V(A_2)=\frac{11}{6}$, $V(A_3)=\frac{4}{3}$, $V(A_0)=\frac{31}{6}$

$F(A_1)=\frac{5}{2}$, $F(A_2) = 2$, $F(A_3) = =\frac{1}{2}$, $F(A_0) = = 4$

By (15) we have

$$\begin{aligned}\frac{10}{3}x-\frac{1}{6}x^2+0x^3&=\frac{7}{2}\\ 2|x|+\frac{11}{6}|x^2|+\frac{4}{3}|x^3|&=\frac{31}{6}\\ \frac{2}{3}|x|+2|x^2|+\frac{1}{2}|x^3|&=4\end{aligned} \quad (20)$$

By solving system (20), we have the exact solution as x = -1.

CONCLUSION

In this article, we introduced a new ranking fuzzy numbers method by LR type fuzzy numbers to solve fuzzy polynomial equation that is transformed to a system of crisp polynomial equations. The transformed system is easily solvable for finding the real roots of the system. In this work the ranking method based on the three parameters like the Real number value, vagueness and Fuzziness is utilized.

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